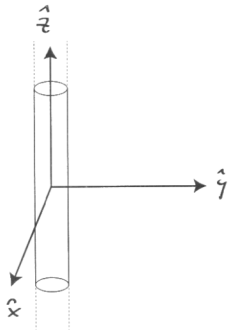
EXAM 15-06-2016

ELECTRICITY AND MAGNETISM 1
#QUESTIONS: 4, #POINTS: 100

WRITE YOUR NAME AND STUDENT NUMBER ON EVERY SHEET. USE A SEPARATE SHEET FOR EACH PROBLEM. WRITE CLEARLY. USE OF A (GRAPHING) CALCULATOR IS ALLOWED. FOR ALL PROBLEMS YOU HAVE TO WRITE DOWN YOUR ARGUMENTS AND THE INTERMEDIATE STEPS IN YOUR CALCULATIONS.

QUESTION 1 - 25 POINTS

Consider an infintely long conducting tube, with radius R, oriented along the $\hat{z}$ axis.



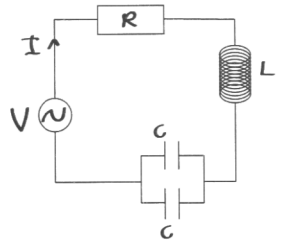
1.A - 5 points. Suppose a surface current density $\vec{K} = K\hat{\phi}$ per unit length. Use Amperian loops to find the magnetic field on the inside and outside of this tube.

1.B - 10 points. Now we fill this tube completely with a paramagnetic material (with an associated linear magnetic susceptibility χ_m). Use the auxiliary field $\vec{H}$ and the symmetry of the problem to find the new value for the magnetic field inside.

1.C - 10 points. As a next step we wind a conducting wire around the filled tube, with n windings per unit length. Find the expression for the current I in this wire that is required to restore the magnetic field inside the tube to its original value (i.e. without the paramagnetic medium). What is the direction of the current in the wire with respect to the surface current?

QUESTION 2 - 25 POINTS

Consider an AC circuit with a resistor R and a coil with an inductance L in series with a parallel pair of equal capacitors, with a capacitance C each.



2.A - 5 points. Give the expression for the complex current in this circuit, making use of the complex impedances for a coil of $Z_L = i\omega L$ and for a single capacitor $Z_C = -i/\omega C$.

2.B - 10 points. Derive the expression for the resonance frequency of this circuit.

2.C - 10 points. Suppose one of the capacitors is filled with a dielectric medium with an electric susceptibility χ_e. How is the expression for the resonance frequency modified?

Question 3 - 25 points

Consider a current loop, with radius R and current I. The current loop is lying flat in the x-y plane, with its center at (0,0,0).

3.A - 5 points. Give the expression for the magnetic dipole moment $\vec{m}$ associated with this current loop, and explain how this dipole moment is different from that of a perfect dipole.

3.B - 5 points. A small section of a large loop can be approximated as an infinitely long straight wire. Use an Amperian loop to find the expression for the magnetic field at a distance $d \ll R$ from the wire.

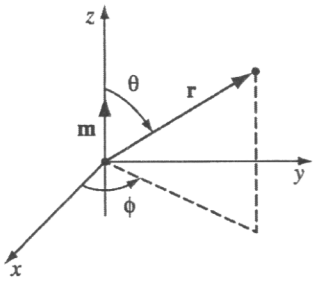
3.C - 5 points. Just above the first loop, at a distance $d \ll R$, we place an identical second current loop. Find (using the approximation from question 3.B) the force on this loop if the current in both loops is flowing in the same direction.

3.D - 10 points. The vector potential for a perfect dipole, located at the origin, pointing in the positive z-direction, is

$$\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}.$$

Show that the expression for the magnetic field of this perfect dipole is as follows:

$$\vec{B}_{dip}(\vec{r}) = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$$



Question 4 - 25 points

4.A - 5 points. Calculate the electric field inside and outside a uniformly charged shell of radius R, with total charge Q.

4.B - 5 points. Calculate the electric field inside and outside a uniformly charged solid sphere of radius R, with total charge Q and a corresponding (uniform) charge density ρ.

4.C - 10 points. Show that the potential inside this uniformly charged solid sphere of radius R, with total charge Q and a corresponding (uniform) charge density ρ is given by

$$V(r) = \frac{Q}{4\pi\epsilon_0}\frac{1}{2R}(3 - \frac{r^2}{R^2})$$

4.D - 5 points. Show that the total potential that you found for a charged solid sphere is larger than that of a charged shell. What are the consequences of this, if the charges of the solid charged sphere were free to move, like in a conductor?

The End